THE POWER OF AN ACTIVELY MANAGED PORTFOLIO: AN EMPIRICAL EXAMPLE USING THE TREYNOR-BLACK MODEL

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ABSTRACT

The focus of this thesis is to examine the added benefits of actively managing a portfolio of securities from an individual investor's perspective. More specifically, managing a market portfolio with the combination of a selected few actively managed securities can, in some instances, create excess return. The active portfolios are formed based on the firms' specific industries or region in which they operate. The idea is that an investor can forecast that a specific industry will outperform or underperform other industries during different periods in the market. Using the investor's forecasts can provide excess returns if the forecast is accurate. On the other hand, investors can hold beliefs about local companies and use those beliefs to forecast firm performance. The logic here is that an investor in his or her local region may have more knowledge about a local company's performance with the notion that company information is more readily available to locals compared to remote investors.

I collected data on the securities that make up the S&P 500 from CRSP. I then made separate portfolios based on the location of the company headquarters and the company industry. I followed a formulation model derived by Jack Treynor and Fischer Black (1973). The purpose of this model is to show how combining a market portfolio with an actively managed portfolio consisting of a few securities can create excess return if predicted returns are correct. If the combined portfolio, a portfolio of selected mispriced securities and the market index, result in an increased slope of the

Capital Allocation Line when compared to the CAL of the market portfolio, then the actively managed portfolio has created an alpha return.

My findings show that the implementation of this model for an individual investor is not plausible. I found that creating accurate forecasts of security prices must involve a team of skilled security and economic analysts. Using historical price returns for my empirical testing provided no definite pattern and therefore I believe that empirical testing may have produced superior results if I forecasted security prices and returns during some prior period and analyzed my accuracy with the portfolio model discussed in this thesis for today's price returns.

ACKOWLEDGMENT

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I. Introduction

A “portfolio” has many definitions that range from a “flat case for carrying documents or drawings” to “securities held by an investor”(Webster Dictionary, 2015). For the purpose of defining “portfolio” in the context of this thesis, we will assume that a portfolio is a collection of securities held by an investor. These investments, or financial assets, constitute shares of companies (sometimes called equities), fixed income securities, commodities (such as oil, wheat, corn, etc.), derivatives (options, futures, forwards, etc.), mutual funds, and other various complex financial instruments. Investors and portfolio managers concentrate their efforts on achieving the best possible trade-off between risk and return. For a portfolio constructed from a fixed set of assets, the risk/return profile varies with the portfolio composition. Portfolios that maximize the return, given the risk, or, conversely, minimize the risk for the given return, are termed optimal portfolios in that they define a line in the risk/return plane called the efficient frontier (Roychoudhury, 2007).

Active investors buy and sell investments in order to exploit profitable conditions. On the other side, passive investors purchase investments with the intention of long-term appreciation with limited turnover. Active and passive investments can serve different needs in the same portfolio. Though most evidence suggests that passive management outperforms active management, some studies suggest that truly active and skilled managers can and do generate returns above the market net of fees (Goldman Sachs, 2010).

The objective of this paper is to study active management by way of seeking alpha, the financial term for excess return. In order to do so, I will gather daily and monthly stock return data for 100 companies that are members of the S&P 500. I will create portfolios based on the company headquarters geographical location. Regional economies throughout the United States respond differently to macroeconomic, and even microeconomic, events. Investors can alter their stock portfolio to encompass the effects of economic swings in a way that may create excess returns. I will make another set of portfolios that are specifically based on a company's industry. Industries either outperform or underperform the market every year. If an investor holds a higher percentage of stocks in an industry that outperforms the market then he or she may create an excess return for his portfolio. Using empirical testing, I will test whether or not a portfolio formed through an active management model will be able to generate a pattern of consistent alpha returns. If this study finds that active management provides returns over that of the market, I will then study the effect of the biased portfolios, in terms of regional or industry construction, have on providing excess returns.

Before studying active management directly, I will explain the basic concepts of portfolio management theory, as these concepts are crucial in the understanding of advanced portfolio models. This section will introduce the concepts of risk and return, the effect of correlation between assets, and the process of introducing risk aversion to the creation of an optimal portfolio that lies on the efficient frontier.

II. Portfolio Management Theory

When investing in a company's stock, investors expect return in the form of dividends or capital gains, or both. The stock return at any time, r_t, is simply the sum of dividends, D_t, and the capital gains, $(P_t - P_{t-1})$, relative to the stock price at time P_{t-1}. Return, r_t, is given by:

$$r_t = \frac{D_t + (P_t - P_{t-1})}{P_{t-1}}$$

In the portfolio context, the expected return of a portfolio, $E(r_p)$, is the weighted average of the expected returns on the individual assets in the portfolio, with weights being the percentage of the total portfolio invested in each asset.

$$E(r_p) = w_a E(r_a) + w_b E(r_b) + \cdots w_n E(r_n)$$

$$E(r_p) = \sum_{i=1}^{N} w_i \, E(r_i)$$

Risk, from a financial point of view, is a statistical measure of the dispersion of outcomes around the mean of expected returns. Portfolio risks can be calculated, like calculating the risk of a single investment, by taking the standard deviation of actual returns of the portfolio over time or by projecting the expected risk based on the probabilities of expected returns. Standard deviation, as applied to investment returns, is a quantitative statistical measure of the variation of specific returns relative to the average of those returns. The variance, σ_p^2, and standard deviation, σ_p, for a portfolio consisting of assets a and b is expressed, respectively, as

$$\sigma_p^2 = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \rho_{ab}\sigma_a\sigma_b$$

$$\sigma_p = \sqrt{\sigma_p^2}$$

In general, portfolio standard deviation will be less than the weighted average of standard deviations of the individual assets in the portfolio. Each individual asset has an expected return and a level of risk associated with holding the asset for a period of time. In the context of a portfolio, holding many assets can, and many times will, greatly diversify risk across the entire portfolio of assets. Diversification is the epitome of “not putting all your eggs in one basket,” but instead investing across a number of assets to reduce risk (Roychoudhury, 2007).

Covariance is the statistical measure of how one asset’s returns in relation to another asset’s. The covariance of a two-asset portfolio is simply the product of the two deviations: the deviation of the returns of security A from its mean, multiplied by the deviation of the returns of security B from its mean (Elton, Gruber, Brown, Goetzmann, 2014). If both assets are increasing in value at the same time or decreasing in value at the same time, they are said to have a positive covariance, and regardless of which way the asset’s returns move, if they move in a parallel fashion the product of the two deviations results in a positive number. The opposite is true for assets that move inversely to each other and is called a negative covariance. Because many times the product of deviations can result in a large number, the covariance can be simplified (or normalized) to a correlation coefficient, which like the covariance, measures the degree of correlation between the two assets.

$$\rho_{ab} = \frac{\sigma_{ab}}{\sigma_a \sigma_b}$$

Dividing by the product of the two standard deviations does not change the properties of the covariance, rather it scales the covariance to have a value between -1≤ ρ_{ab} ≥1. Intuitively, +1 is perfect positive correlation in that assets *a* and *b* move in direct proportion to each other. Conversely, -1 is a perfect negative correlation in that assets A and B move in negative proportion to each other.

Another key concept in optimizing one's portfolio is utility theory. A utility function measures an investor's relative preference for different levels of expected return (Norstad, 1999).

$$U = E(r) - \frac{1}{2} A\sigma^2$$

A is a measure of risk aversion, which is measured as the marginal reward that an investor requires to accept additional risk. More risk-averse investors require greater compensation for accepting additional risk. Thus, *A* is higher for more risk-averse individuals.

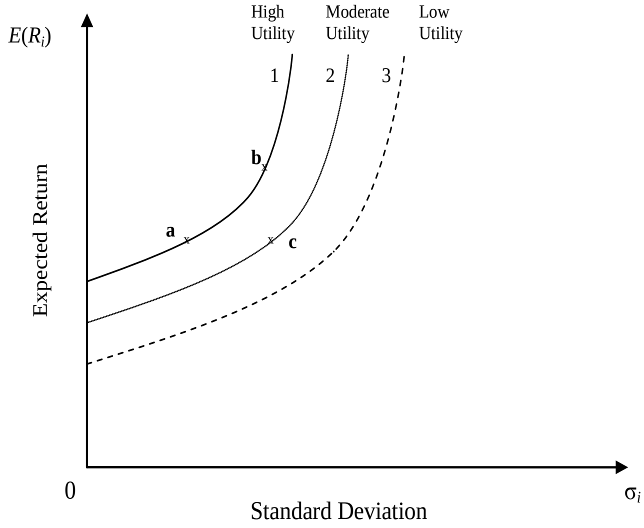

Figure 1 Indifference Curves (CFA Institute)

Several conclusions can be drawn from the utility function in Figure 1. First, utility is unbounded on both sides. It can be highly positive or highly negative (CFA Institute). Second, higher return contributes to higher utility (CFA Institute). Third, higher variance, and thus higher standard deviation, reduces the utility but the reduction in utility is amplified by the risk aversion coefficient, *A* (CFA Institute). Utility can always be increased, albeit marginally, by getting higher return or lower risk. Fourth, utility does not indicate or measure satisfaction (CFA Institute). It can be useful only in ranking various investments. For example, a portfolio with a utility of 4 is not necessarily two times better than a portfolio with a utility of 2. The portfolio with a utility of 4 could increase our happiness 10 times or just marginally. By definition, all points on any one of the three curves have the same utility. Referring to Figure 1, an investor does not care whether he or she is at point *a* or point *b* on indifference curve 1. Point *a* has lower risk and lower return than point *b*, but the utility of both points is the same because the higher return at point *b* is offset by the higher risk.

Like indifference curve 1, all points on indifference curve 2 have the same utility and an investor is indifferent about where he or she is on curve 2. When comparing point *c* with point *b*, point *c* has the same risk but significantly lower return than point *b*, which means that the utility at point *c* is less than the utility at point *b*. Given that all points on curve indifference 1 have the same utility and all points on indifference curve 2 have the same utility and point *b* has higher utility than point *c*, indifference curve 1 has higher utility than indifference curve 2. Therefore, risk-averse investors with utility functions represented by indifference curves 1 and 2 will prefer indifference curve 1 to curve 2. The utility of risk-averse investors always increases as you move northwest-higher return with lower risk. Because all investors prefer more utility to less, investors want to move northwest to the indifference curve with the highest utility.

Another important concept is mondern portfolio theory is the efficient frontier, shown in Figure 2, which models the risk-return trade off. The frontier is depicted in a graphic form as a curve comparing portfolio risk against the expected return.

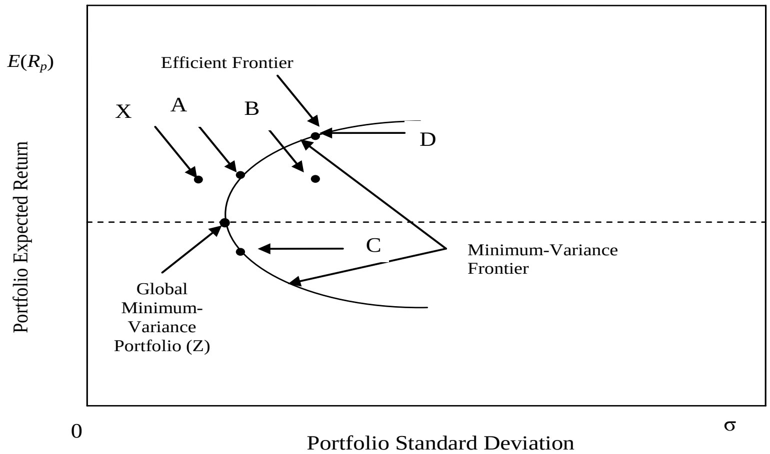

Figure 2 The Efficient Frontier (CFA Institute)

Every possible asset combination can be plotted in risk-return space, and the collection of all such possible portfolios defines a region in this space. The portfolios that have the least risk for each possible level of return are known as the minimum variance frontier. The curve (from z rightward) along the upper edge of this region is known as the efficient frontier. Combinations along this line represent portfolios with the lowest risk for a given level of required return or the highest required return for a given level of risk. Conversely, for a given amount of risk, the portfolio lying on the efficient frontier represents the combination offering the best possible return. Consider points A, B, and X in Figure 2 and assume that they are on the same horizontal line by construction. Thus, the three points have the same expected return, $E(r_1)$, as do all other points on the imaginary line connecting A, B, and X. Given a choice, an investor will choose the point with the minimum risk, which is point X. Point X, however, is unattainable because it does not lie within the investment opportunity set. Thus, the minimum risk that we can attain for $E(r_1)$, is at point A. Point B and all points to the right of point A are feasible but they have more risk.

Therefore, a risk-averse investor will choose only point A in preference to any other portfolio with the same return.

Before proceeding further, we must introduce a risk-free asset, r_f. A risk-free rate is the rate one can earn by investing in risk-free assets such as Treasury bills or money market funds. Treasury bills are determined to be a riskless investment because: 1) Treasury bills are the original issue discount instruments, 2) that are short-term (maturity at issue of one year or less), and 3) Treasury bills are issued by the U.S Treasury Department and thus, investors believe that the U.S government will not default on payments. We know that the set of investment possibilities created by all combinations of risky and risk-free assets is the Capital Allocation Line (CAL). An investor can vary the amounts allocated to the risky portfolio and risk-free asset to move along the CAL. This is an important concept for a risk-averse investor. The CAL represents a line tangent to the minimum-variance frontier at the investor's desired risk/return trade off point and is calculated by:

$$E(r_p) = r_f + \left[\frac{r_i - r_f}{\sigma_i}\right] \sigma_p$$

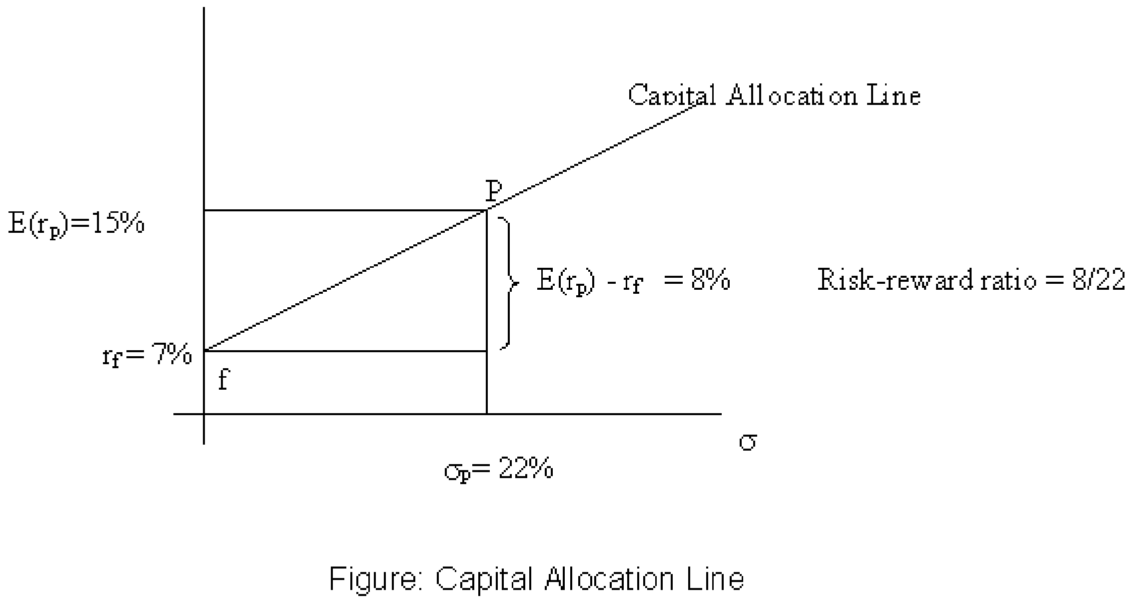

Figure 3 Capital Allocation Line

Points under the preferred CAL may be attainable, but are not preferred by any pragmatic investor because the investor can get a higher return for the same risk by moving to an asset located on the CAL. Points above the CAL are desirable but not achievable with available assets.

William Sharpe introduced the Sharpe ratio, also known as the reward-to-volatility ratio, as the average return in excess of the risk-free rate per unit of volatility or total risk (Bodie, Kane, & Marcus, 2010). By adding the risk-free asset, investors can choose a portfolio that increases the Sharp ratio (increased risk-premium for given amount of risk) while still maintaining a position along the efficient frontier. Graphically, as seen in Figure 4, the portfolio with maximum Sharpe ratio (point *P*) is the point where a line through the origin is tangent to the efficient frontier, in mean-standard deviation space, because this point has the property that has the highest possible mean-standard deviation ratio. The Sharpe ratio is calculated by:

$$S_i = \frac{E(r_i) - r_f}{\sigma_i}$$

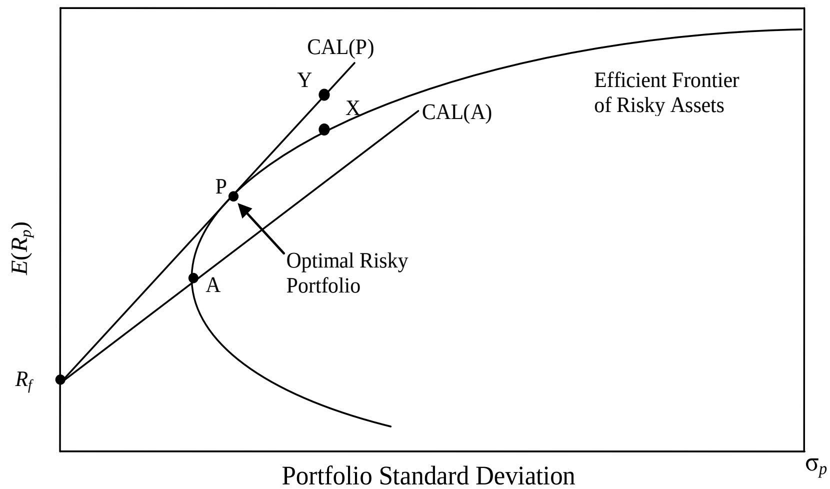

Figure 4 Capital Allocation Line and the optimal risky portfolio (CFA Institute)

When the CAL is combined with the efficient frontier, we can mathematically determine the one portfolio that would be preferred by all pragmatic investors. In theory, we can have as many CALs as we have portfolios along the efficient frontier, however, only one of these CALs is preferred. Refer to points P and A located on the efficient frontier in Figure 3. Both points can be combined with the risk-free asset to form a CAL. Pragmatic investors will prefer the CAL that combines the risk-free asset with portfolio P [CAL(P)] to the CAL that passes through portfolio A [CAL(A)] as all points along CAL(P) yield a higher rate of return for a given level of risk than the points along CAL(A). The CAL that passes through a portfolio on the efficient frontier and provides the optimal risk-return trade-off is the CAL, and hence portfolio, that all investors would prefer. These statistical concepts or measures are the centerpiece for any portfolio optimizing method. Every investor has a set of preferences and objectives that are used to construct his or her optimal portfolio. To simplify and help visualize the way a portfolio can be constructed, I will use a simple model of a portfolio containing two risky assets with normally distributed returns. Again, this model assumes that the investor is risk averse, meaning that if there are two assets with identical returns, the investor will prefer the less risky asset.

III. The Risk-Return Trade Off With Two Risky Assets

Assume the two risky assets, *A* and *B*, are available for consideration in an investment portfolio. Also assume there are no transaction costs or taxes. A risk-free asset in the form of U.S Treasury bills allows borrowing and lending at the risk-free rate. The portfolio return is as follows:

$$r = w_a r_a + w_b r_b$$

The asset weights (or proportions) need to add up to one:

$$w_a + w_b = 1$$

The expected return equals:

$$E(r_p) = w_a E(r_a) + w_b E(r_b)$$

Portfolio variance is:

$$\sigma_p^2 = w_a^2 \sigma_a^2 + w_b^2 \sigma_b^2 + 2 w_a w_b \rho_{ab} \sigma_a \sigma_b$$

Simplified to a standard deviation of:

$$\sigma_p = \sqrt{w_a^2 \sigma_a^2 + (1 - w_a)^2 \sigma_b^2 + 2 w_a (1 - w_a) \rho_{ab} \sigma_a \sigma_b}$$

Now, assume that $\rho_{ab} = 1$, implying that assets A and B are perfectly positively correlated. We know this indicates perfect correlation to each other, thus implying there are no gains to be had from diversification. The opposite is true for $\rho_{ab} = -1$, where A and B are perfectly negatively correlated. With this type of correlation, a perfect hedging opportunity is presented as diversification benefits are maximized (Bodie, Kane, & Marcus, 2011). An investor can reduce portfolio risk simply by holding instruments that are not perfectly correlated. In other words, investors can reduce their exposure to individual asset risk by holding a diversified portfolio of assets. Diversification allows for a weighted average portfolio return with reduced risk.

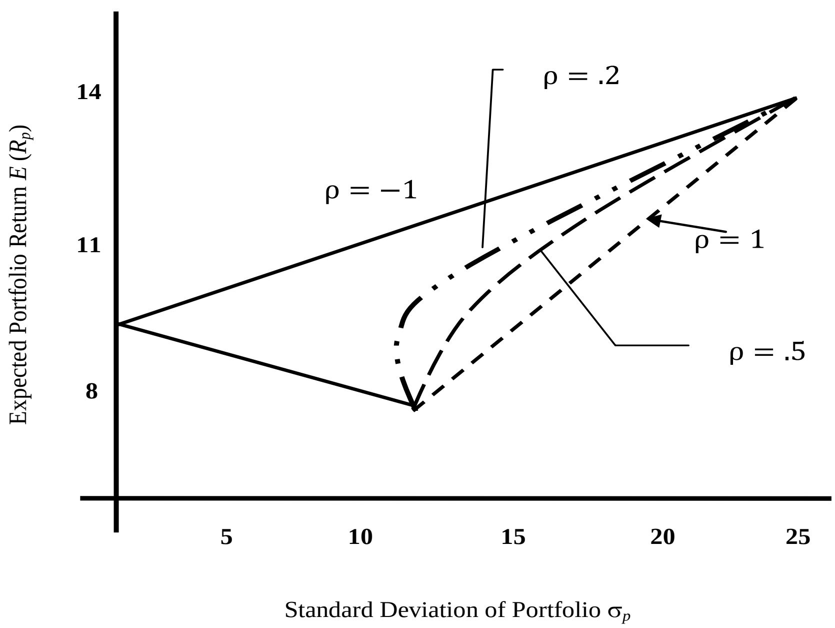

Figure 5 Relationship between expected return and standard deviation of return for various correlation coefficients (CFA Institute)

Figure 5 depicts the relations between the expected return and standard deviation of returns for portfolios of two stocks with various correlation coeffecients. The uncurved dashed line where correlation between assets is 1 indicates there is no benefit to diversification. The solid line represents a correlation of -1. When this is the case, all risk can be eliminated by investing a positive amount in the two stocks. Because most assets are not perfectly correlated, portfolio combinations of most multi-asset portfolios will lie on a curve that curves to the left. Thus, as the correlation becomes smaller, as it approaches zero, the curve becomes more defined as the benefit of diversification pushes the curve northwest given that a smaller correlation coefficient reduces the portfolio standard deviation.

For two risky assets, we know that the various portfolios curve to the left in an expected return/standard deviation graph if they are less than perfectly correlated.

The concepts discussed in the preceding pages are important in understanding the concepts of portfolio theory. I will now explain the background of how these concepts can be interpreted by introducing Harry Markowitz's Modern Portfolio Theory.

IV. Modern Portfolio Theory

Prior to Harry Markowitz's 1952 "Portfolio Selection" article in the *Journal of Finance*, the process of using diversification in holding securities was a well-established practice, but lacked an adequate theory. Markowitz formally established the effects of diversification when risks are correlated, distinguished between efficient and inefficient portfolios, and analyzed risk-return trade-offs for the portfolio as a whole (Markowitz, 1952). By formalizing the concept of diversification, Markowitz proposed that investors should focus on selecting portfolios based on their joint risk-reward features instead of merely compiling individually attractive securities regardless of their relation to the other securities in their portfolios (Markowitz, 1952). The Modern Portfolio Theory maintains that the "essential aspect pertaining to the risk of an asset is not the risk of each asset in isolation, but the contribution of each asset to the risk of the aggregate portfolio" (Royal Swedish Academy of Sciences, 1990). The expected return of a portfolio is a weighted average of the returns on the individual securities and the variance of return on the portfolio is a particular function of the variances of, and the covariance between, securities and their weights in the portfolio. Furthermore, Markowitz proposed that means, variances, and covariance of securities be estimated by a combination of statistical analysis and security analyst judgment. Using the estimates found by these analytical models, the set of efficient mean-variance combinations can be derived and presented to an investor for choice of the desired risk-return combination (Markowitz, 1952). This practice became known as the Modern Portfolio Theory (MPT).

Uncertainties about future economic events make economic indicators unpredictable and cause turbulence in financial markets. The criticism of the MPT is that the theory focus’ on highly complex statistics-based mathematical modeling and formulas that are not easily calculated. The theory requires mathematical calculations on expected values, based on past performance to measure the correlations between risk and return. However, past performance is not a guarantee of future performance and thus, taking into account only past performances is frequently misleading.

Markowitz portfolio selection assumes the market is efficient, thus meaning, the mean and variance of data represent the true performance of those assets. A shortfall of this assumption is the MPT relies on asset prices making it vulnerable to various market vagaries such as environmental, personal, strategic, or social investment decision dimensions.

Realizing the shortcomings of his theory due to the complexity of the computational procedures and amount of input data needed to perform portfolio analysis, Markowitz became interested in simplifying the portfolio selection problem. His original mean-variance analysis presented difficulties in implementation: to find a mean-variance efficient portfolio, one needs to calculate the variance-covariance matrix with N(N- 1)/2 elements. Thus, a reasonably sized portfolio of 100 securities requires the daunting task calculating 4,950 variances and covariances.

$$\frac{100(100-1)}{2} = 4{,}950$$

Markowitz's Modern Portfolio Theory is a valuable tool to learn as a basis for portfolio construction theory, but implementation of this theory in a strict sense is not practical, because to build an efficient portfolio for an investor we need to know the expected returns, expected variances and expected covariances of all possible candidates for inclusion in the portfolio. Although the Markowitz portfolio theory has provided a fundamental breakthrough towards strengthening the mean-variance analysis framework, modifications, extensions and alternatives to the theory have been formed to simplify and prioritize assumptions of the theory and to address the limitations of the framework.

V. Capital Asset Pricing Model

William Sharp, John Lintner, Jan Mossin, and Jack Treynor developed the Capital Asset Pricing Model (CAPM) to simplify the insights of Markowitz's Modern Portfolio Theory (MPT). The CAPM predicts the relationship between risk and equilibrium returns on risky assets (Bodie, Kane, Marcus, 2010). Sharpe (1964) and Lintner (1965) add two key assumptions to the Markowitz model to identify a portfolio that must be mean-variance efficient. The first assumption is complete agreement: given market clearing asset prices at *t*-1, investors agree on the joint distribution of asset returns from *t*-1 to *t* (Fama & French, 2004). The second assumption is that there is borrowing and lending at a risk-free rate, which is the same for all investors and does not depend on the amount borrowed or lent (Fama & French, 2004). CAPM takes into account an asset's sensitivity to non-diversifiable risk (systematic risk) while being held in a well-diversified portfolio. The expected return of an asset is driven by its systematic risk, β_i, which indicates how much, on average, the stock return changes for each additional 1% change in the market return. Beta is calculated as the covariance between an asset and the market return divided by the variance of the market return as follows:

$$\beta_{iM} = \frac{cov(R_{i,}R_M)}{\sigma^2(R_M)}$$

Therefore, the regression of the rate of return on the individual security i is shown by:

$$r_i = r_f + (r_M - r_F)\beta_i + \alpha_i + \epsilon_i$$

Because the market beta of asset i is also the slope in the regression of its return on the market return, a common interpretation of beta is that it measures the sensitivity of the asset's return to variation in the market return (Fama & French, 2004). A larger value of beta implies greater financial risk since beta reflects volatility in expected returns compared to the market. The expected return on any asset i is the risk-free interest rate, r_f, plus a risk premium, which is the asset's market beta, β_{iM}, times the premium per unit of market risk, $E(r_M) - r_f$.

$$E(r_i) = r_f + [E(r_M) - r_f]\beta_{iM}$$

This equation tells us that the expected return on an individual security is determined by the risk-free rate, the market risk premium, and beta (Nam, 2011). The fact that there is no residual excess return explains that investors should hold the market portfolio under the assumption that all investors have the same expectations and the market is perfectly efficient. As a result, in this paper, we can use the expected return on the individual stock from the CAPM as a benchmark return and the market portfolio as a benchmark portfolio in order to measure residual return and risk.

The assumption that investors care only about the mean and variance of distributions of one-period portfolio returns is extreme. Perhaps investors also care about how their portfolio return covaries with labor income and future investment opportunities, so a portfolio's return variance will miss important dimensions of risk (Fama & French, 2004). If so, market beta is not a complete description of an asset's risk, and we should not be surprised to find that differences in expected return are not completely explained by differences in beta. In the late 1970's research began to uncover variables like market capitalization, various price ratios, and momentum that add to the explanation of average returns provided by market beta.

VI. Active Management

Through the previous sections, we look over the portfolio theories and, under perfect capital markets, the active portfolio management does not survive as all investors invest their money in a combination of risk-free asset and the market portfolio, which has the highest expected return given the level of risk depending on an investor's indifferent curve. However, the empirical test of this thesis will aim to find patterns that capture alpha returns by altering individual security weights in the market portfolio that reflect outperformance or underperformance by constraints of a company’s region or industry. In this section, we define the active portfolio with residual return (alpha), risk, and information ratio.

We begin with the definition of active portfolio management:

> *Active portfolio management is the implementation of a dynamic investment strategy that over-and underweights the predefined investment opportunities over a long-term basis, with the single objective of outperforming the predefined benchmark at a predefined time in order to add value to the portfolio (Nam, 2011)*

Commonly applied benchmarks in active portfolio management are large and highly liquid indices such as the S&P 500 or the Dow Jones Index. The S&P 500 is a market-value-weighted index and is comprised of the largest 500 market capitalization companies in the United States. Because the index is made up of many companies it would be hard for an investor to purchase each individual security that comprise the index. In order to diversify assets without buying each security in the S&P 500, an investor can invest in an Exchange Traded Fund (ETF). An ETF tracks the overall index but quantifies the index into an asset or share that can be bought or sold. The advantage of this particular approach is that the benchmark's underlying assets are likely to follow a somewhat similar return pattern as the overall market, making it less difficult to allocate portfolio assets. The SPDR (SPY) is an S&P 500 ETF Trust that seeks to provide investment results that correspond generally to the price and yield performance of the S&P 500 Index (SPDR.COM). For the purpose of this thesis, we will us the (SPY) as a passive benchmark with which to compare our returns of active management.

The key concept of the active portfolio construction is how to organize the residual alpha and risk from the alpha generating strategy into the current portfolio. Even though Markowitz's mean-variance portfolio optimization model is the starting point for the portfolio construction, this model is not quite applicable for investors due to the input sensitivity. In the next section I introduce the active asset allocation method used in this thesis.

VII. Treynor-Black Model

The presumption of market efficiency is inconsistent with the existence of a vast industry engaged in active portfolio management. Jack Treynor and Fischer Black (1973) proposed a model to construct an optimal portfolio with respect to this assumption, when security analysts forecast abnormal returns on a limited number of covered securities (Kane, Kim, White, 2003). We will refer to this method as Treynor Black (TB).

The purpose of the TB is to maintain the overall quantitative framework of the efficient markets approach to portfolio selection while simultaneously introducing a critical violation of the efficient markets theory: individual portfolio managers may possess information about the future performance of certain securities that is not reflected in the current price or projected market return of the asset. Because inefficiently priced securities have forecasted alpha returns, Treynor and Black attempt to explore and identify such mispriced securities to add to a passive index portfolio. In order to do so, there must be a method to measure these abnormal returns, thus the quantitative performance measure for a single asset used in in this model is alpha (α), the projected return of the security over-and-above its market risk-adjusted return. In constructing this portfolio, the forecasted alpha securities are added to a diversified market portfolio to provide a return greater than what a portfolio invested solely in the index fund would return. The optimal portfolio would then "tilt" towards securities with projected outperformance (alpha greater than zero) and away from securities with projected underperformance (alpha less than zero). The efficiency of the model depends critically on the ability to predict alpha returns. It follows that security analysts must submit quantifiable forecasts subjected to continuous and rigorous testing to evaluate the individual performance pertaining to the return over that of the market (Kane, Kim, White, 2003).

The optimal portfolio must be a mix of the covered securities and the index portfolio that results in a new tangency portfolio along the Capital Allocation Line (CAL) (Bodie, Kane. & Marcus, 2011). Securities not covered by the analyst that make up the index portfolio are assumed to be priced efficiently as the active portfolio analyst can only cover a small number of securities that are believed to be inefficiently priced, thus the reason for seeking alpha. TB identifies the portfolio of only the covered securities (Active Portfolio, A) that can be mixed with the index (Passive Portfolio, M) to obtain the optimal risky portfolio.

The initial weight of each security in the active portfolio should be proportional to the expected alpha return of the individual security, (α_i), divided by the unsystematic risk squared, $(\sigma^2(e_i))$, where the unsystematic risk is the volatility in the security's price, which is not due to macroeconomic factors.

$$\frac{\alpha_i}{\sigma^2(e_i)}$$

By way of this formula, we can assign initial weights to securities in the active portfolio and then scale these weights in a way such that the higher alpha of the security, the higher the weight assigned to the security. This scaling is also used in measuring volatility in that the higher the volatility of security's price, due to firm-specific risk, the lower the weight assigned in the active portfolio. For a negative alpha, we can expect a negative weight in the active portfolio, representing a short position. The new scaled positions that form the new active portfolio weights must sum to 1 and is shown by:

$$w_i = \frac{\alpha_i / \sigma^2(e_i)}{\sum_{i=j}^{n} \frac{\alpha_j}{\sigma^2(e_j)}}$$

Treynor and Black measure the added benefits of seeking alpha by way of the ratio of the portfolio alpha to the portfolio specific risk (nonsystematic risk). The portfolio alpha is the weighted average of the alpha for each asset, using the share in the portfolio as the weight, and the portfolio specific risk (square root of the portfolio variance), where the portfolio variance is the weighted sum of the asset-specific risks squared. We add specific risk together in this manner because it is, by definition, independent from asset to asset (Miller, 1999).

$$\alpha_A = \sum_{i=1}^{n} w_i \, \alpha_i$$

$$\beta_A = \sum_{i=1}^{n} w_i \, \beta_i$$

$$\sigma^2(e_A) = \sum_{i=1}^{n} w_i^2\, \sigma^2(e_i)$$

After computing the alpha and residual standard deviation of the active portfolio we can determine the weight of the active portfolio in the overall portfolio. This model requires that the weight of the active portfolio should be:

$$w_A^0 = \frac{\alpha_A/\sigma_A^2}{(E(r_M) - r_f)/\sigma_M^2}$$ [1]

It is possible that the beta for the active portfolio exhibits high systematic risk or a high beta. In order to avoid having the overall portfolio be too risky, a correction can be made to have the weight of the active portfolio scaled further in such a way that the beta of the active portfolio does not change the beta of the overall portfolio. By doing so, an active portfolio with a large beta will be reduced to a smaller weight in the overall portfolio in order to have the original beta of the passive portfolio remain unchanged upon mixing the active and passive portfolios.

$$w_A^* = \frac{w_0}{1 + (1 - \beta_A)w_0}$$

Once the weight of the adjusted active portfolio is calculated, the weight of the passive portfolio can be found by subtracting the adjusted weight of the active portfolio from one.

$$w_M = 1 - w_A^*$$

[1] Note: $E(R_M)$ represents both the expected return on the market and return on passive portfolio.

The combination of the two weights must sum to 1 and represent the percentage of each portfolio that will be combined to form the overall optimal portfolio.

Once the new weights are assigned to both active and passive portfolios, the risk-premium, $E(r_P)$, for the new combined portfolio is calculated by:

$$E(r_p) = (w_M + w_A^* \beta_A) E(r_M) + w_A^* \alpha_A$$

And the variance for the combined portfolio is be calculated by:

$$\sigma_{cp}^2 = (w_M + w_A^* \beta_A)^2 \sigma_M^2 + [w_A^* \sigma(e_A)]^2$$

To illustrate the performance of the new optimal risky portfolio, the Sharpe ratio of the passive, or market, portfolio, which measures the slope of the Capital Allocation Line, is added to the Information ratio of the active portfolio. The Information ratio measures the residual return to residual risk. The two ratios combined should produce a new Capital Allocation Line with a steeper slope, thus representing a higher expected return while maintaining the amount of risk equal to the passive portfolio.

The Sharpe ratio for the passive portfolio, M, is shown by:

$$S_M = \left[\frac{E(r_M) - r_f}{\sigma}\right]^2$$

The Information ratio of the active portfolio, which is also the Sharpe ratio of the active portfolio, is shown by:

$$I = \left[\frac{\alpha_A}{\sigma_A}\right]^2$$

The Capital Allocation Line for the new optimal portfolio, P, consisting of both the active and passive portfolios, includes the sum of active and passive Sharp ratios.

$$S_{cp} = \left[\frac{E(r_M) - r_f}{\sigma_M}\right]^2 + \sum_{i=1}^{n} \left[\frac{\alpha_A}{\sigma_A}\right]^2$$

If the two ratios added together result in a new Capital Allocation Line with a steeper slope than that of the passive portfolio's Capital Market Line, then the addition of the active portfolio will result in a new efficient frontier that has a higher expected return for the same level of risk. That is, $(r_{cp} - r_f)/\sigma_p$ > $(r_M - r_f)/\sigma_p$. This points out the motivation of the Treynor Black model: an actively managed portfolio covering only a limited number of securities can be added to an already diversified market equilibrium portfolio, and will provide an added alpha premium return over the market risk premium for the market portfolio.

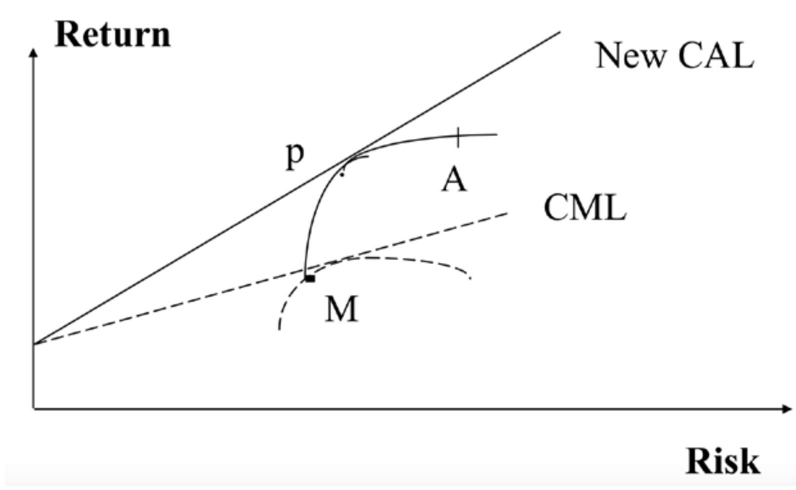

Figure 6 The efficient frontier moves upwards from point M to point P because of the alpha return

Figure 6 accurately depicts the positive effects of active management as the new combined portfolio results in a CAL with a steeper slope than the market portfolio.

VIII. Empirical Example

The purpose of this thesis is to explain active portfolio management, but also to empirically test the active management model derived by Treynor and Black to construct separate active portfolios in combination with the benchmark. More specifically, the empirical section of this thesis will use publicly available financial market information to implement the Treynor-Black model. Although we do not have private information to test empirically, we assume that it is possible some investors have such information and can therefore exploit price inefficiencies. Intuitively, if an investor can exploit price inefficiencies to create alpha with only public information, then an investor who holds private information will undoubtedly be able to do the same.

Financial research has yielded a large number of in-depth studies concerning the investments by professional money managers, yet historically, relatively little has been known about individual investors' money management, in no small part because of the shortage of reliable, high quality data available for academic research (Ivkovic & Weisbenner, 2005). In the world we live in, individual investors have many channels of finding quality information about a company, including, for example, media coverage, analyst valuation, and quarterly and yearly earning reports, in order to form opinions regarding particular companies. With this knowledge, one could hypothesize that investors could gather relative and valuable information about companies local to them with greater ease than they could about remote companies that have little effect on an investor's local economy. If investors, in fact, believe they have information about a company or a specific market sector that is not reflected in the current market price, then the investor can use that superior knowledge to enhance portfolio return beyond simply investing in the market.

The next section of this chapter will relate to the data retrieved from the Center for Research in Security Prices (CRSP) and FederalReserve.gov to form portfolios for providing excess return for both an active portfolio invested solely in either location or industry biased constraints. More specifically, the aggregate data taken from the CRSP was on companies that made up the S&P 500. I gathered monthly returns for each of the securities in the index from 2000-2014 and then aggregated the months into yearly returns. I then chose companies with which I was familiar with to analyze the statistical components and annual returns of each security to derive my active portfolio. The return on the S&P 500 was used as the annual return on the market. The risk-free rates were taken from the Federal Reserve website. I selected one regional-based and one industry-based portfolio to use for illustrative purposes. These portfolios use the annual return for the year 2013. The portfolio return charts can be seen in the Appendix of this thesis as well as the tables holding the list of selected securities that make up both the region portfolio and industry based portfolio.

VIII-1. Data

Referring to the table of selected securities in A-1, and the portfolio return chart in A-2, we can see that the active section of the combined portfolio provided a return over that of the market. We can see for each complete portfolio, that is, the active and market portfolio combined, with the identified weights set forth in the model, provide returns over that of market or index fund.

Looking at the table of selected securities in A-3, and the portfolio return chart in A-4, we can see, once again, that the active section of the combined portfolio provided return greater than the return on the S&P 500. This portfolio may have some insight into the original hypothesis of this thesis in that investors may be able to predict a certain industry will outperform or underperform the market. In the case of portfolio A-4, three of the five securities included in the consumer discretionary industry have negative alphas and thus negative weights in the portfolio. As an investor, if I could forecast that these companies were to underperform the market in a given year, I could increase my overall return by selling these securities short.

The risk-free rate was essentially zero as the portfolios modeled in Appendix A-2 and A-4 tracked the annual returns during 2013 (when Treasury bill rates were almost zero). Thus, the risk premium, $(r_M - r_f)$, is almost equal to the expected return, and thus the CAL is going to be rather steep in slope.

Portfolio A-2, and A-4, identifies the basic logic for Treynor-Black model in that excess return is possible if forecasts are accurate. The combination of the active and passive portfolios allow an investor who seeks to manage his or her money in an aggressive way the ability to potentially create returns over the market. Conversely, the market portfolio allows an investor the security of not investing all his or her money in an allocation method that may or may not play out, depending on the accuracy of the forecasts.

IX. CONCLUSION

I formed 20 separate region-based portfolios and 20 separate industry-based portfolios for each year, 2000-2014 with the original notion of finding patterns in creating excess return by actively managing a combination of passive and active portfolios. I performed the analysis on all 40 portfolios. However, given that portfolio formation was based on historical information I used the prior years data to determine portfolio allocations rather than projections, the outcomes were not always feasible. For example, many portfolios had an extreme amount of leverage and were therefore not ultimately included in the study. Rather, I am showing a couple portfolios for illustrative purposes to show that the model can work.

The work analyzed in this thesis supports the basis of active management and the Treynor-Black model, in that it makes sense that an analyst could perhaps analyze a few stocks allowing the formation of superior opinions regarding the future of those securities, thus allowing the weights of the active portfolio to represent the opinions set forth. When identifying securities that are mispriced, whether over or under, analysts can use the knowledge or opinions they hold about the mispriced security to create an active portfolio to mix or combine with a market portfolio so that not all of an investor’s money is invested in the riskier active portfolio. Although it appears my study has analyzed returns that successfully support the Treynor-Black model, there are some shortcomings to these successes. For one, I was unable to identify any real pattern between excess returns for the active portfolios that were based on either region or industry, largely due to the fact that I had to use past performance as a forecast for the returns. As historical data allowed me to create inputs for the formation of my portfolios, it also hindered my hypothesis in that no real forecasting of industry or regional based performance occurred.

Accurate forecasting is not an easy process-even for skilled security analysts. Researching active portfolio management with the Treynor-Black model has proven to me that success with this model is very forecast dependent, meaning successful implementation of this model is critically dependent on analysts successfully picking and forecasting accurate returns. In this thesis, we used historical data as a forecasting tool on the selected securities. As discussed in the preceding pages of this thesis, past performance is not a prediction of future performance.

Illustrating this model is the easy part as the financial and statistical concepts are consistent with the vast amount of portfolio optimization model in the world, however, after researching this method of active management, I can firmly state that this method is not easy for an individual investor to implement. To actively and successfully implement this method, it would take the work of a team of security and economic analysts to come up with the inputs (forecasts) for the mispriced securities and for the market as a whole. I simply do not believe many individual investors have of the economic and financial knowledge to efficiently identify and act upon the mispricing's of such securities in a consistent and reliable manner.

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APPENDIX

A-1. Portfolio based on Region

Companies in this region portfolio are all located in Northern California. Regional portfolio formation is based on the belief in that an investor could hold opinions regarding companies that affect their local economies over that of a remote investor.

Security	Ticker Symbol	GICS Sector	Headquarters
Wells Fargo	WFC	Financials	San Francisco, California
Charles Schwab Corp.	SCHW	Financials	San Francisco, California
eBay Inc.	EBAY	Information Technology	San Jose, California
Cisco Systems	CSCO	Information Technology	San Jose, California
Altera Corp	ALTR	Information Technology	San Jose, California

A-2: Combined portfolio returns for the combined active and passive portfolios based on region.

- All of the results in this table are from the regression with $(r_i - r_f) = \beta(r_m - r_f) + e_i$

Ticker Symbol	Standard Deviation (σ)	Beta (β)	Alpha (α)	Annual Return	Risk-Free Rate
ALTR	0.26399	1.41199	-0.18474	-0.03735	0.000583333
CSCO	0.21955	1.07069	-0.00025	0.16491	0.000583333
EBAY	0.26443	1.25378	0.48872	0.09679	0.000583333
SCHW	0.18398	1.46657	0.10450	0.85499	0.000583333
WFC	0.13919	1.32204	0.09273	0.33539	0.000583333
S&P 500	0.08991	1.00000	-	0.28683	

	S&P 500	Active Portfolio A	ALTR	CSCO	EBAY	SCHW	WFC	Combined Portfolio P
$\sigma^2(e)$			0.06969072	0.04820222	0.06992322	0.03384864	0.01937385	
$\alpha/\sigma^2\,(e)$		12.20695886	-2.65085501	-0.00518648	6.98938014	3.08727318	4.78634710	
w_i			-0.21715933	-0.00042487	0.57257341	0.25291091	0.39209987	1.0000000
α_A		0.38273481	0.04011801	0.00000010	0.27982808	0.02642919	0.03635942	
$\sigma^2(e_A)$		0.03135382	0.00328648	0.00000000	0.02292365	0.00216509	0.00297858	
w_A^0		0.34403205	0.34403205	0.34403205	0.34403205	0.34403205	0.34403205	
w^*	0.61636189	0.38363810	0.38363810	0.38363810	0.38363810	0.38363810	0.38363810	
Beta (β)		1.30008266	-0.30662681	-0.00045491	0.71788109	0.37091156	0.51837172	1.11512314
Risk-Premium	0.28624666	0.75487914	0.00823757	-0.00006981	0.05508537	0.21608877	0.13127765	0.67519278
Standard Deviation (σ)	0.08991000	0.17707112	0.0032864873	0.0000000087	0.0229236523	0.0021650922	0.0029785815	0.10967461
Return	0.17679108	0.15775310						33.454418%

A-3: Portfolio based on Industry

Companies in the industry portfolio are represented in the consumer discretionary sector. An investor could hold opinions based on the cyclical or economic cycles that affect specific industries and act upon those opinions in hopes of outperforming the market.

Security	Ticker Symbol	GICS Sector	GICS Sub Industry
Polo Ralph Lauren	RL	Consumer Discretionary	Apparel, Accessories & Luxury Goods
Tiffany & Co.	TIF	Consumer Discretionary	Apparel, Accessories & Luxury Goods
Bed Bath & Beyond	BBBY	Consumer Discretionary	Specialty Stores
NIKE Inc.	NKE	Consumer Discretionary	Apparel, Accessories & Luxury Goods
The Walt Disney Co.	DIS	Consumer Discretionary	Broadcasting & Cable TV

A-4: Combined portfolio returns for the combined active and passive portfolios based on industry.

- All of the results in this table are from the regression with $(r_i - r_{rf}) = \beta(r_m - r_{rf}) + e_i$

Ticker Symbol	Standard Deviation (σ)	Beta (β)	Alpha (α)	Annual Return	Risk-Free Rate
BBBY	0.27092	1.19901	-0.13631	0.45418	0.000583333
DIS	0.13491	0.94528	0.20723	0.55094	0.000583333
NKE	0.21279	0.68898	0.01761	0.54454	0.000583333
RL	0.22618	1.32054	-0.04484	0.19665	0.000583333
TIF	0.25546	1.11299	-0.20576	0.64830	0.000583333
S&P 500	0.08991	1.00000	-	0.28683	

	S&P 500	Active Portfolio A	BBBY	DIS	NKE	RL	TIF	Complete Portfolio P
$\sigma^2(e)$			0.07339764	0.01820070	0.04527958	0.0511573	0.06525981	
$\alpha/\sigma^2(e)$		5.88814721	-1.8571440	11.38582075	0.38891699	-0.8765106	-3.1529358	
w_i			-0.3154038	1.93368479	0.06605082	-.14886018	-0.5354716	1.0000000
α_A		0.56172688	0.04299269	0.40071750	0.00116315	0.00667489	0.11017864	
$\sigma^2(e_A)$		0.09539959	0.00730156	0.06805493	0.00019754	0.00113361	0.01871193	
w_A^0		0.16594725	0.16594725	0.16594725	0.16594725	0.16594725	0.16594725	
w^*	0.84185603	0.15814396	0.15814396	0.15814396	0.15814396	0.15814396	0.15814396	
Beta (β)	1.00000000	0.70265854	-0.3781723	1.82787356	0.04550769	-0.1965758	-0.5959745	.95297724
Risk-Premium	0.28624666	0.76286055	-0.1430661	1.06421631	0.03592878	0.02918651	-0.3468339	.35209969
Standard Deviation (σ)	0.08991000	0.30886000	0.0085544	0.26087341	0.01405495	0.03366919	0.1367915	.14977610
Return	0.24146956	0.09198315						33.345272%