WHAT DID YOU EXPECT? HOW EXPECTATIONS AND THE STRUCTURE OF STUDENT LOANS AFFECT THE INCENTIVES TO PURSUE AN UNDERGRADUATE EDUCATION

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ABSTRACT

Student loan debt and the cost of a college degree are on the rise each year, even as the value of a college degree appears to be stagnant. Yet, little research is available on how the characteristics of student loans create the incentives that have caused this effect. In this paper, I create a two-period, inter-temporal model for examining two related questions. First, what effect do interest rates and the structure of student loans have on individuals’ incentives to attend college? Second, how do expectations of graduating high school students about future earnings impact their decision to attend or not attend college? I hope this will begin a discussion on the programs currently available to students to fund a college education. Ultimately, with greater knowledge on how current programs fair in achieving their stated goals of promoting greater educational attainment throughout society, increasing social welfare, lessening the burden of student loan debt, and retaining revenue neutrality in government programs, we can hope for and expect more informed decisions by policy makers on this important topic.
# TABLE OF CONTENTS

List of Tables and Figures................................................................. vi

Chapter 1: Overview........................................................................... 1
  1.1. Literature Review ................................................................. 2
  1.2. Loan Types ........................................................................... 5
  1.3. The Value of a College Degree .............................................. 9
  1.4. Summary ............................................................................. 11

Chapter 2: Model ............................................................................. 13
  2.1. Basic Model .......................................................................... 13
  2.2. Example of Basic Model ...................................................... 15
  2.3. Expanded Model ................................................................... 16
  2.4. Example of Expanded Model ............................................... 19
  2.5. Cost of Attendance ............................................................... 20
  2.6 Summary ............................................................................. 21

Chapter 3: Treatments .................................................................... 22
  3.1 Low Interest Rate Loans ....................................................... 22
  3.2 Expectations ......................................................................... 25
  3.3 Sensitivity Test ...................................................................... 26
  3.4 Summary ............................................................................. 28

Chapter 4: Conclusion ..................................................................... 29
  4.1 Discussion ............................................................................ 30
  4.2 Areas for Further Research ................................................... 31

Appendices ....................................................................................... 34
  Appendix A – Equations 1, 2, and 8 ........................................... 35
  Appendix B – Equation 3 ............................................................ 36
  Appendix C – Equation 4 ............................................................ 37
  Appendix D – Equation 5 ............................................................ 38
  Appendix E – Equation 6 ............................................................ 39
  Appendix F – Equation 7 ............................................................ 40

Citations ........................................................................................ 41
List of Tables and Figures

Table 1 - Loan Types.................................................................................................................. 7
Figure 1.1 – Earnings Premium of a College Degree ................................................................. 10
Figure 2.1 – Example of Basic Model .......................................................................................... 16
Figure 2.2 – Example of Expanded Model .................................................................................. 20
Table 2 – Sensitivity Test.............................................................................................................. 27
Chapter 1: Overview

Student loan debt is an ever-present issue for over 43 million Americans, who have amassed nearly 1.3 trillion dollars of outstanding debt. Year-to-year, these debt numbers are increasing, with the graduating class of 2016 accumulating 6% more debt than the class of 2015 according to nonprofit loan refinancing company Student Loan Hero.

The effects of this are far reaching. The incentives to attend college are diminished, leading to fewer college attendees and a less educated populace. Many individuals with high amounts of student debt are forced to declare bankruptcy, occasionally multiple times, putting the American dream and financial security further and further out of their reach. Taxpayers and lenders fear the impending liability should this debt become so great and so risky as to be entirely unmanageable.

For these reasons, the issue of student loan debt has never been more important. It should come as no surprise that newly elected President Trump included in his campaign platform an income-based repayment proposal to combat growing student loan debt. Other candidates for that office, such as Senator Rubio, Senator Sanders, and Secretary Clinton discussed their own proposals during the 2016 presidential campaign.
Given the importance of this issue, more analysis of the potential options is needed. In this paper, I begin by reviewing the present research on the topic and discussing the policy options available for addressing student loan debt. Next, I construct a simple theoretical model that can be used to evaluate these policies. I then use the model to evaluate two research questions. First, what effect do interest rates and the structure of student loans have on individuals’ incentives to attend college? Second, how do the expectations of graduating high school students about future earnings affect their decision to attend or not attend college? My model does not evaluate the actual decisions of real people, but instead seeks to understand how interest rates and expectations affect the theoretically optimal decision. Finally, I summarize my findings and what they can tell us about these questions.

1.1. Literature Review

In analyzing the effects of student debt and student loan types, two important aspects are the different loan programs and prospective students’ expectations on the costs and benefits of attending college. It is worth reviewing the present literature on student loan debt, the cost of college, the role of government in paying for college, and the effects of such on students. This creates a context from which the results of the model I present can be analyzed. Without an understanding of the ramifications of debt loads and government involvement, it is impossible to determine the deeper findings of the model.

Is a college degree valuable? I discuss this more in depth in section 1.3, but it is worth noting here that the answer is a qualified “yes.” The qualifier is the
assumed risk. Evidence suggests that, “…enrolling in college is equivalent to signing up for a lottery with large expected gains – indeed...college is a better investment today than it was a generation ago – but it is also a lottery with significant probabilities of both larger positive, and smaller or even negative, returns” (Avery and Turner 2012, 188). In her paper, Susan Dynarski writes, “The typical student holds debt that is well below the lifetime benefits of a college education” (2014, 25).

While I found numerous articles from mainstream media sources such as the Wall Street Journal and Economist speculating on a student debt bubble, i.e., where the cost of a college degree has risen above its actual value, the academic literature I came across only found the value of a college degree to still be substantially greater than the cost, such as the two cited above. Aside from these two examples, the paper by Daly and Bengali that I discuss in section 1.3 serves as a third such example.

However, that a degree has value doesn’t necessarily mean that it is a good investment for prospective students or taxpayers. There are student loan programs that have insurance against micro and macroeconomic shocks but these are not revenue neutral. Other programs are revenue neutral but lack insurance. The former benefits borrowers but is costly to taxpayers, while the latter negatively impacts borrowers but has no tax burden. There is academic literature suggesting it is possible to design a program that can insure borrowers against micro and macroeconomic shocks, while remaining revenue neutral, but only if more data on student borrowing were available (Dynarski 2014). As long as such data are unavailable, arguments can be made for and against the cumulative value of government student loan programs.
Arguments have been made for some time regarding the subsidization of higher education, assuming a revenue neutral program is difficult or infeasible to implement. On the one side, Milton Friedman wrote in 1955 that education’s primary purpose is to create citizenry capable of participation. Past this, government should only make funds available, “…not as a subsidy but as ‘equity’ capital. In return, the student would obligate himself to pay the state a specified fraction of his earnings above some minimum, the fraction and minimum being determined to make the program self-financing,” (16). This is likely one of the earliest arguments in favor of an income-based or pay-as-you-earn payment plan for undergraduate debt. On the other side, some argue this same level of responsibility to its citizens by a government is justification for subsidization. “The state owes citizens the right to an education adequate enough for them to be involved in making decisions that affect them… this might mean nothing more revolutionary than free tuition repaid by progressive taxes, or it might mean professional education repaid by national service for a period of time” (Engel 1984, 32). This debate may be resolved as we gain more knowledge and can create more agreement on the effects of student debt and increased subsidies on social welfare.

Lochner and Monge-Naranjo claim that subsidized student loans are highly inefficient, arguing that money would be much better used for grants or reduced tuition. Further, it may be that by subsidizing student loans with tax receipts there is actually a net decrease in social welfare (2015). However, evidence shows that this may not necessarily be the case. Wigger points out that the structure of the tax code can have important implications on government’s ability to affect social welfare
through education subsidies, such as subsidized undergraduate student loans (2004). In this discussion it is important to also recognize the social welfare cost of students who choose to forgo graduate level degrees due to burdensome undergraduate debt. Data show “Students with debt of $5,000 or higher are significantly less likely to apply to graduate or first professional school than their peers who did not have educational debt” (Millett 2003, 19).

Ultimately it appears that much more research is needed. While the literature I encountered suggests with some confidence that an undergraduate degree is valuable, it is still debatable to what level these degrees ought to be subsidized, if at all, through student loan programs. Additionally, it is not clear what the effects of overly burdensome student debt are on social welfare.

In this paper, I seek to expand the current research through a study of how different loan types influence the incentives to pursue an undergraduate degree. My approach seeks to shed additional light on this important issue, and serve as a useful tool to policy makers trying to achieve a wide array of complex objectives.

1.2. Loan Types

The Stafford loan is the most common federal student loan available to incoming undergraduate students. These loans are split into subsidized and unsubsidized varieties based on financial need. These loans can be for a maximum of $57,500 over a student’s four years, but with different limits each year based on a student’s classification and dependency status. The interest rate is fixed and tied to
the 10 year U.S. Treasury bill, determined at the time of disbursement. Students can defer any payment until graduation.

Far less common but preferable to Stafford loans are Perkins Loans. These loans are similar to Stafford loans, but typically have a lower interest rate and each undergraduate institution establishes its own eligibility requirements. Each loan program has its own federal funds pool. The size of this pool for Perkins loans is about one tenth that of the pool for Stafford loans. Because Perkins loans are far less available than Stafford loans, the limits on these loans, set by each university as stated, tend to be lower. For many students who qualify for Perkins loans, it makes sense to combine it with some level of Stafford loan in order to fully fund their education.

The common characteristics of Perkins and Stafford loans, shown in the first two rows of Table 1, are that they target low-income families, offer a discounted interest rate compared with typical consumer loans, and require no credit history. According to Student Loan Hero, a student loan debt management company, these two low-interest rate loan types account for the majority of new and outstanding student loans, 78.2% of debtors and 55.4% of total debt as of 2012.

There are also other unique refinancing options for students. These options, shown in the third through fifth rows of Table 1, collectively account for $439.2 billion of total student loan debt (34% of all debt), but only 12 million borrowers (27% of debtors). The most prevalent of these options is loan forgiveness. This typically takes the form of forgiveness in exchange for service. For example, anyone working in public service for ten years can have his or her federal student debt
forgiven. Similarly, one can have student debt forgiven on a state-by-state basis for pursuing careers in education, medicine, law, or other fields in high demand.

Table 1 shows data on loan types available to undergraduates.

**Table 1 – Loan Types**

<table>
<thead>
<tr>
<th>Loan Type</th>
<th>Interest Rate</th>
<th>Eligibility</th>
<th># Debtors (% total)</th>
<th>Outstanding Debt (% Total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stafford</td>
<td>Tied to 10 yr. treasury note; 3.76% in 2016/17</td>
<td>Based on proven financial need through FAFSA</td>
<td>31.9 Million (72.2%)</td>
<td>$690.3 Billion (54.8%)</td>
</tr>
<tr>
<td>Perkins</td>
<td>5%</td>
<td>Based on proven financial need through FAFSA</td>
<td>2.7 Million (6.1%)</td>
<td>$8 Billion (0.6%)</td>
</tr>
<tr>
<td>Private</td>
<td>Varies</td>
<td>Varies by Lender</td>
<td>1.4 Million (3.2%)</td>
<td>$6.2 Billion (0.5%)</td>
</tr>
<tr>
<td>Income-based*</td>
<td>Varies, based on annual income</td>
<td>Varies</td>
<td>3.1 Million &gt; 5.27 Million (7% &gt; 11.9%)</td>
<td>$269 Billion (21.3%)</td>
</tr>
<tr>
<td>Parent PLUS**</td>
<td>Tied to 10 yr. treasury note; 6.31% in 2016/17</td>
<td>Based on Credit Score</td>
<td>3.3 Million (7.5%)</td>
<td>$74.5 Billion (5.9%)</td>
</tr>
<tr>
<td>Total</td>
<td>N/A</td>
<td>N/A</td>
<td>44.2 Million (100%)</td>
<td>$1.26 Trillion (100%)</td>
</tr>
</tbody>
</table>

*Refinancing that includes all repayment plans that are based on your income, including income-contingent (ICR), Income-based (IBR), Pay as You Earn (PAYE), and Revised Pay as You Earn (REPAYE) plans.

**Loan is taken out and paid back by parents rather than by student.

Source: Data originates from the Federal Student Aid Office in the United States Department of Education. Student Loan Hero, a student loan debt-refinancing corporation, has consolidated the data for use here.
Another option gaining popularity is income-based or pay-as-you-earn loans. Under these schemes, students pay a percentage of their discretionary income, typically between 10% and 15%, over the course of a predetermined amount of years, usually between 10 and 25 years. At the end of the time period, any remaining debt is forgiven.

Finally, some students choose private loans. These can come in a variety of forms but typically are accompanied by a higher interest rate than federal loans. Perhaps for this reason, they are far less popular, accounting for less than half of one percent of all student loan debt. The primary benefit of private loans is typically higher limits. It is not uncommon for students to mix private loans with federal loans.

There are many small programs for student borrowing and student loan refinancing not represented in the table. These programs are too small in nature to make discussion worthwhile, but together make up the remaining 16.9% of student debt not presented in Table 1.

The three primary types of federal student loans are low-interest rate loans, income based repayment schemes, and loan forgiveness. Although numerous options have been outlined above, these three types of loans account for 99% of student loan debt. With my model, I will primarily focus on low-interest rate loans, which make up a majority of student loans and student loan debt, for answering my first research question. In chapter 4, I comment on how my model can be extended to analyze the characteristics of income based repayment schemes and loan forgiveness.
1.3. The Value of a College Degree

In analyzing the incentives to attend college, it is important to gain an understanding of the differences in earnings that a college degree provides over a high school diploma. While there may be an intrinsic value from a degree, for my purpose I assume the primary benefit is the increase in expected lifetime earnings. Understanding the value of a college degree will be especially important in answering the second research question, how do expectations of graduating high school students about future earnings affect the incentives to attend or not attend college?

Especially helpful in discussing the value of a college degree is the work of Mary C. Daly and Leila Bengali. In their paper, “Is it Still Worth Going to College?” they analyze the value of an undergraduate degree relative to a high school degree. They begin by analyzing the “earnings premium” of college graduates over high school graduates, finding that between 1968 and 2011, college graduates earned a premium of between 41% and 61% per year. Daly and Bengali then broke these data down into three cohorts (groupings by 20 year graduation intervals), finding that this earnings premium has consistently risen with each new cohort and the more years individuals are past graduation. This is shown graphically in Figure 1.1.

Their work illustrates that for the 1950s-1960s cohort a college graduate could expect to earn $20,000 more annually by the time they were ten years past graduation. This premium decreased slightly for the 1970s-1980s cohort before rising significantly for the 1990s-2000s cohort. Additionally, their study found that
the range for this premium over the period studied was from a lifetime average of 43% greater annual income in 1980 to 61% greater annual income in 2011. This consistent and significant earnings premium is credited to the value of a college degree, and the improved ability of college graduates to avoid unemployment, especially during economic downturns.

![Graph showing earnings premium of a college degree](image)

**Figure 1.1 – Earnings Premium of a College Degree**

Source: Reproduction of Daly and Bengali’s figure, pg. 2; their text: “PSID and authors’ calculations. Premium defined as difference in mean annual labor income of college graduates in each year since graduation and earnings of high school graduates in years since graduation plus four. Values are three-year centered moving averages of annual premiums.”

Finally, Daly and Bengali utilize a discounted future earnings model to determine break-even values for attending and not attending college. Their ultimate conclusion is that the value of a college degree still exceeds the cost. Daly and Bengali find that the average college graduate pays off their degree by 20 years
post-graduation, and then continues to earn dividends throughout the remainder of
their working life so that they have substantially higher lifetime earnings relative to
high school graduates. Ultimately, the average college graduate in their study could
expect to make over $830,000 more in lifetime earnings, after subtracting the cost of
tuition and opportunity cost of attendance.

It is clear that a college degree had significant value into the beginning of the
21st century. However, in recent years the increasing cost of attendance combined
with the increased cost from incurring significant college-related debt threatens to
erode this value. For example, it is possible that many college graduates who incur
such debt will be unable to pay off this debt 20 years post-graduation and will never
begin to accrue the dividends of a college degree. Additionally, students who fail to
properly account for the full costs of a college degree or overestimate future
earnings from a college degree may falsely believe attendance is their optimal
choice. This is what I seek to evaluate using my own model in the following
chapters.

1.4. Summary

Student loan debt is a significant issue in modern American life. While the
value of a college degree is, in fact, on the rise, so are the costs. For students who can
afford to pay for college without accruing debt, there are obvious and clear benefits.
However, for many Americans, who don’t have such an opportunity, it is less
obvious if the value of a college degree outweighs the full costs.
In chapter 2, I present my model for examining this central issue. This model allows an analysis of the incentives to attend or not attend college for students who do require student loans. In chapter 3, I manipulate the model to address the two central research questions. First, what effect do interest rates and the structure of student loans have on individuals’ incentives to attend college? Second, how do the expectations of graduating high school students about future earnings affect their decision to attend or not attend college? In chapter 4, I conclude by summarizing my findings, conclusions, and areas for further research.
Chapter 2: Model

In this chapter, I present the model that I use to analyze the questions enumerated in Chapter 1. First, what effect do interest rates and the structure of student loans have on individuals’ incentives to attend college? Second, how do the expectations of graduating high school students about future earnings affect their decision to attend or not attend college?

I begin by describing the basic two-period inter-temporal consumption model, followed by a numerical example of its application. I then expand the model by including alternate incomes for the choices to attend college or not attend college, and include an example of its application. Finally, I complete the model by incorporating cost of attendance. The goal of this section is to clarify the model and begin to evaluate its robustness for analyzing the research questions.

2.1. Basic Model

I present a two-period, inter-temporal consumption model for assessing the decision to attend or not attend college. This Cobb-Douglas utility function is depicted in equation 1, where $C_1$ represents consumption in the first period (period 1) and $C_2$ represents consumption in the second period (period 2). In equation 1, $C_1$ has an exponent of 1, while $C_2$ has an exponent of $b$, where $0 < b < 1$. The exponent $b$
must be less than 1 but greater than 0 to show the relative preference for present consumption over future consumption.

\[ U(C_1, C_2) = C_1 * C_2^b \]  \hspace{1cm} (1)

Second, I construct a budget constraint for this individual, whereby \( m_i \) represents the individual’s first period income, \( m_2 \) represents the individual’s second period income, \( r \) represents the market interest rate, and \( C_1 \) and \( C_2 \) again represent period 1 and period 2 consumption, respectively. The budget constraint is depicted in equation 2; the right hand side is total consumption, the left hand side is total income. The constraint is that total consumption cannot exceed total income. The period 2 values \( C_2 \) and \( m_2 \) are shown as discounted present values, i.e., divided by \((1+r)\) to value future amounts in present value.

\[ C_1 + \left( \frac{C_2}{1+r} \right) = m_1 + \left( \frac{m_2}{1+r} \right) \]  \hspace{1cm} (2)

Next, I evaluate the individual’s optimal consumptions for \( C_1^* \) and \( C_2^* \) by maximizing utility, given the budget constraint. The problem, its constraints, and its solution are shown in equation 3. Appendix B shows the derivation of equation 3.

\[ U(C_1, C_2) = C_1 * C_2^b \text{ subject to } C_1 + \left( \frac{C_2}{1+r} \right) = m_1 + \left( \frac{m_2}{1+r} \right) \]  \hspace{1cm} (3a)

\[ C_1^* = \left( \frac{1}{1+b} \right) \left( \frac{m_2}{1+r} + m_1 \right) \]  \hspace{1cm} (3b)

\[ C_2^* = \left( \frac{b}{1+b} \right) \left[ m_1 (1+r) + m_2 \right] \]  \hspace{1cm} (3c)

\[ U^* = C_1^* * C_2^{b*} \]  \hspace{1cm} (3d)
Equation 3a depicts the constrained utility maximization problem. Equations 3b and 3c depict the optimal period 1 and period 2 consumption, respectively. Equation 3d shows the maximum utility $U^*$ that the individual can achieve given the budget constraint.

Equations 3b and 3c allow evaluation of whether the individual is a borrower or lender in period 1. If $C_1^* > m_1$, then the individual is a net borrower. Likewise, if $C_1^* < m_1$, the individual is a net lender. The relationship between $C_2^*$ with $m_2$ is inverse to the relationship of $C_1^*$ with $m_1$. Because a borrower in the model must repay principal and interest, borrowing in period 1 will make $C_2 < m_2$ and lending in period 1 will make $C_2 > m_2$. This will be important in chapter 3 where I evaluate the individual’s decision to attend or not attend college, and how different loan programs affect this decision.

2.2. Example of Basic Model

For the purposes of example, I use values that allow easy computations and portray the model’s basic intuitions. That stated, I use values of $b=.5, m_1=$ $50, m_2=$ $55$, and $r=.1$ (i.e., a 10% interest rate). Using these values in equations 3b and 3c yields optimal consumptions $C_1^*=$ $66.67$ and $C_1^*=$ $36.67$. This person would be a net borrower of $C_1^*-m_1=$ $16.67$, with a utility of $U^*=$ $404$.

This simple example illustrates how a person makes choices about current and future consumption, given each period’s income and the interest rate. When alternative income streams are introduced – income with a college degree and
without – this model becomes an interesting tool for evaluating the choice to attend college.

An example of the basic model is shown in figure 2.1.

**Figure 2.1 – Example of Basic Model**

The curved indifference curve represents the utility function. The straight line represents the budget constraint. The optimal choices $C_1^*$ and $C_2^*$ occur where the two are tangent. This is the highest utility the individual can achieve and still meet his or her budget constraint.

### 2.3. Expanded Model

In order to actually evaluate the decision to attend or not attend college, it becomes important to expand the model to include the individual’s choice between two different income streams. If the individual does not attend college, he works full-time in periods 1 and 2. This income stream is denoted by $m_1w$ and $m_2w$. If the
individual goes to college, he works part-time in period 1 while attending college and works full-time in period 2. This income stream is denoted by $m_1^c$ and $m_2^c$.

I will make some assumptions regarding incomes. When an individual chooses to attend college, he receives a relatively low wage in period 1. However, in period 2, he will receive a relatively high wage as a result of his human capital investment from period 1. When an individual chooses to not attend college, he will receive a moderate wage in both period 1 and period 2, but his period 2 wage is higher than his period 1 wage because of gained experience. However, his wage in neither period will be as great as that of the period 2 wage for the choice to attend college. I also assume that the period 1 wage for the choice to attend college is less than the period 1 wage for the choice to not attend college, and that the period 2 wage for the choice to attend college is greater than the period 2 wage for the choice to not attend college. Lastly, I assume the combined wages from attending college are greater than the combined wages from not attending college. These assumptions are described in equations 4a-4e, which imply equation 4f. Appendix C shows the derivation of equation 4.

\begin{align*}
    m_1^c &< m_2^c \quad (4a) \\
    m_1^w &< m_2^w \quad (4b) \\
    m_1^c &< m_1^w \quad (4c) \\
    m_2^w &< m_2^c \quad (4d) \\
    m_1^w + m_2^w &< m_1^c + m_2^c \quad (4e) \\
    m_1^c &< m_1^w < m_2^w < m_2^c \quad (4f)
\end{align*}
These assumptions allow the creation of two different budget lines, one for when the individual does not attend college and one for when he does, for which the utility function can be maximized. I draw on the utility maximization problem from equation 3 for each case to derive equation 5 and equation 6. The derivations of equation 5 and equation 6 are shown in Appendix D and Appendix E respectively.

Equation 5 represents the choice to not attend college.

\[
\text{Max } U = C_1 \times C_2^b \text{ subject to } C_1 + \left( \frac{C_2}{1+r} \right) = m_1^w + \left( \frac{m_2^w}{1+r} \right) \tag{5a}
\]

\[
C_1^w = \left( \frac{1}{1+b} \right) \left( \frac{m_2^w}{1+r} + m_1^w \right) \tag{5b}
\]

\[
C_2^w = \left( \frac{b}{1+b} \right) [m_1^w (1+r) + m_2^w] \tag{5c}
\]

\[
U^w = C_1^w \times C_2^{wb} \tag{5d}
\]

Equation 6 represents the choice to attend college.

\[
\text{Max } U = C_1 \times C_2^b \text{ subject to } C_1 + \left( \frac{C_2}{1+r} \right) = m_1^w + \left( \frac{m_2^w}{1+r} \right) \tag{6a}
\]

\[
C_1^c = \left( \frac{1}{1+b} \right) \left( \frac{m_2^c}{1+r} + m_1^c \right) \tag{6b}
\]

\[
C_2^c = \left( \frac{b}{1+b} \right) [m_1^c (1+r) + m_2^c] \tag{6c}
\]

\[
U^c = C_1^c \times C_2^{cb} \tag{6d}
\]

The new optimal consumption values for each choice give the utilities of the respective choices. By comparing the utilities $U^w$ and $U^c$, I can determine whether or not the individual will attend or not attend college. If $U^w > U^c$, the individual will work
in period 1. If $U^w < U^c$, the individual will attend college in period 1. An example of this expanded model is provided in section 2.4.

In section 2.5 I incorporate cost of attendance into the model. Using this model, I can manipulate the interest rate and relative incomes to assess how low-interest loans will affect the decision to attend or not attend college. It is also possible to evaluate how false expectations about future earnings can lead to an individual making a choice contrary to his own self-interest. I address both low-interest rate loans and expectations in chapter 3.

2.4. Example of Expanded Model

For the purpose of example, I use values of $b = .5$, $m_1^w = 50$, $m_2^w = 55$, $m_1^c = 5$, $m_2^c = 102$, and $r = .1$. I use equation 5 to compute $C_1^w = 66.67$, $C_2^w = 36.67$, and $U^w = 404$. Using equation 6, I compute $C_1^c = 65.15$, $C_2^c = 35.83$, and $U^c = 390$. In this simplistic example, the individual will choose to work and forgo a college education, despite the fact that his lifetime income is greater for attending college than not attending college ($107$ compared to $105$). This is because the individual puts a relatively high weight on current consumption $C_1$ compared to future consumption $C_2$, i.e., $b = .5$.

It should be noted that if the interest rate were to fall, e.g., to $1\%$ from $10\%$, the individual would choose to attend college. In this example the individual would have utility values of $U^w = 413 < U^c = 422$. Similarly, should the wage gap for period 2 income due to a college degree be increased past a certain threshold, the individual would opt to attend college. In my example with a $10\%$ interest rate, if $m_2^c \geq 105$, the optimal choice will be to attend college.
An example of the expanded model is depicted in figure 2.2.

![Graph showing budget lines and utility functions]

**Figure 2.2 – Example of Expanded Model: The Individual Chooses Not to Attend College**

### 2.5. Cost of Attendance

Finally, in this section I incorporate into the model a cost of attendance, e, in period 1 for the choice to attend college. Intuition tells us that this will have the most direct effect on the model by decreasing net income in period 1 for the choice to attend college.

Including a cost of attendance will modify equation 6, yielding equation 7. Equation 7a is identical to equation 6a, except that $m_1^c$ is replaced by $m_1^c-e$. 
Equation 7 represents the choice to attend college with cost of tuition included. Appendix F shows the derivation of equation 7.

\[
\text{Max } U = C_1 \cdot C_2^b \text{ subject to } C_1 + \left( \frac{C_2}{1+r} \right) = (m_1^c - e) + \left( \frac{m_2^c}{1+r} \right) \quad (7a)
\]

\[
C_1^e = \left( \frac{1}{1+b} \right) \left( \frac{m_2^c}{1+r} + (m_1^c - e) \right) \quad (7b)
\]

\[
C_2^e = \left( \frac{b}{1+b} \right) \left[ (m_1^c - e)(1 + r) + m_2^e \right] \quad (7c)
\]

\[
U^e = C_1^e \cdot C_2^b \quad (7d)
\]

Note that using equation 6 and equation 7, \(C_1^c-C_1^e=e/(1+b)>0\) and \(C_2^c-C_2^e=be/(1+b)>0\). Thus \(U^c>U^e\), and including the cost of attendance lowers the utility of attending college.

2.6 Summary

In chapter 2, I explain the basic model, which is the foundation for the rest of my evaluation. From there, I expand the model to include a decision to attend college or to work full-time in period 1. Finally, I include a cost of attendance for the decision to attend college in period 1. These expansions of the model increase the robustness and applicability of the model.

In chapter 3, I will analyze how different expectations and loan programs change the decision to attend or not attend college, and begin to determine answers to the research questions.
Chapter 3: Treatments

In this chapter I seek to answer the research questions using the model presented. First, what effect do interest rates and the structure of student loans have on individuals’ incentives to attend college? Second, how do the expectations of graduating high school students about future earnings affect their decision to attend or not attend college? These questions are important because their answers can serve as useful tools in evaluating how certain policy options impact the incentives of potential college students deciding whether or not to attend college.

Specifically, in section 3.1, I answer the first research question by looking at the effect of low interest rate loans, judging the effect of allowing students to borrow at a below-market interest rate. In section 3.2, I answer the second research question by looking at the effect of incorrect expectations. In section 3.1, I analyze the sensitivity of the model to different interest rates and coefficients of $b$.

3.1 Low Interest Rate Loans

The key difference between evaluation of low interest rate loans and the base model is the incorporation of a second interest rate $i$ representing the interest rate of borrowing to attend college. Here, I assume $i$ is lower than the market rate of interest $r$, e.g., the interest rate on a student loan is less than the interest rate on a personal consumption loan. Additionally, it should be noted that money borrowed for this purpose can only be used to pay for college, eliminating arbitrage.
opportunities by taking out such loans at the low interest rate then subsequently lending the same funds at the higher market rate. Equation 8 shows the modifications to equation 7 for a student with the opportunity to obtain a student loan at a below-market interest rate. In equation 8, the student’s period 1 income

\[
Max U = C_1 \times C_2^b \text{ subject to } C_1 + \left( \frac{C_2}{1+r} \right) = m_1^c + \left[ \frac{m_2^c - e(1+i)}{1+r} \right]
\]  

(8a)

\[
C_1^t = \left( \frac{1}{1 + b} \right) \left[ m_1^c + \frac{m_2^c - (1 + i)e}{1 + r} \right]
\]

(8b)

\[
c_2^t = \left( \frac{b}{1 + b} \right) \left[ m_1^c(1 + r) \right] + m_2^c - [(1 + i)e]
\]

(8c)

\[
U^i = C_1^t \times C_2^b
\]

(8d)

returns to \( m_1^c \), but his period 2 income is now \( m_2(1+i)e \). That is, the individual defers the attendance cost until period 2, at which time the individual pays back principle plus interest.

To create a useful example of the model, I assume values of \( b = .9333, r = .1 \) (10%), \( m_1^w = $30, m_2^w = $32.89, m_1^c = $15.08, m_2^c = $52.92, e = $3.4, \) and \( i = .09 \) (9%). The values for incomes and cost of attendance are in thousands of dollars per year. To create these assumptions, I use real world data for \( b, m_1^w, m_2^w, m_1^c, \) and \( m_2^c \). These values come from Daly and Bengali’s paper: a value of 6.667% discounted future value, the average value of an AAA bond between 1990 and 2011; a value of $32,890 mean annual income for non-college graduates; and a value of $52,920 mean annual income for college graduates. A Georgetown study found the average college student is able to earn $15,080 annually through part time or full time work (Carnevale et al.
By using real world data for some of the values, the model is made more interesting and robust.

Two values that are not based on real-world data are $e = 3,400$ and $r = 10\%$. This tuition figure is well below the national average of just over $25,000 for annual tuition. Additionally the current market rate of interest for a typical student is closer to 5%. However, using such estimates would unnecessarily complicate the example by creating utility values with differences to multiple decimal places. Because utility values are a theoretical tool used to evaluate optimal decision making, not a real world value and are ordinal, not cardinal, using simplified values allows for an easier understanding of the model’s findings without actually changing any of the intuitions.

Under these assumptions, a potential undergraduate student who must borrow at the market rate of interest to pay the cost of tuition will choose to forgo a college education to work instead: using equations 6 and 8, with $i = r$, I calculate $U^w = 782 > U^e = 780$. However, if that student is eligible for a low interest student loan, he will instead opt to attend college: using equations 6 and 8, I calculate $U^w = 782 < U^i = 785$. This is of interest because it shows how a low interest rate loan can change the optimal choice for a potential college attendee. In the example here, despite only a 1% lower interest rate, the individual’s choice changed from not attending college to attending college.

Further, this example is in line with one’s intuition. It makes sense that borrowing at a discounted interest rate is preferable to borrowing at the market rate of interest. This example is interesting because it shows that discounted
interest rates can change the optimal choice for a recent high school graduate trying to decide whether or not to attend college.

Most importantly, this example allows for a qualified answer to the first research question: what effect do interest rates and the structure of student loans have on individuals’ incentives to attend college? Low interest student loan programs can increase the incentive to attend college for at least some students.

3.2 Expectations

The second research question is how do the expectations of graduating high school students about future earnings affect their decision to attend or not attend college? This is an important question because if a student’s expectations vary from actual results, this may lead him to make a suboptimal decision about attending or not attending college. Further, this would serve as the beginning of an explanation for a bubble in higher education, should one exist.

To address the research question, I will use the expanded model with cost $e$, but for simplicity all borrowing occurs at the market rate of interest $r$. Low-interest rate loans could be included, but would unnecessarily complicate the example without changing any of the intuitions. I again assume $b=0.9333$, $r=0.1$ (10%), $m_1^w=30$, $m_2^w=32.89$, $m_1^c=15.08$, $m_2^c=52.92$, and $e=3.4$. Incomes and cost of tuition are stated in thousands of dollars per year. In this example the individual chooses to work instead of attend college, despite the fact that the lifetime earnings are greater for attending college: $U^w=782 > U^c=780$. However, what if this individual falsely expected to earn more after completing a college degree than the actual $m_2^c$?
To address this, I denote expected period 2 income for a student who attends college as $m_2^x$. I assume $m_2^x = 54$, which is higher than the true $m_2^x = 52.92$. Under such circumstances, the individual would have an expected utility of $U^x = 805$ from attending college, which is greater than the utility from not attending college in period 1, $U^w = 782$. Because the expected utility is greater than that of working, but the realized utility $U^e = 780$ is less than that of working, this individual will make a suboptimal choice to attend college.

This is useful because it shows how even a small difference between expected and annual income, <2%, can lead to an individual making a suboptimal choice. If such a misperception were widespread, a large number of individuals could make a suboptimal decision to attend college. Collectively this could lead to a bubble in the area of higher education. If market participants are willing to pay more for a product than its actual value, that will create a bubble. College degrees are not like other investment instruments in that their value cannot be transferred from one individual to another. This makes it extremely difficult to determine if there is a bubble in the undergraduate education sector. However, with this model, it’s clear how one could be or have been started.

3.3 Sensitivity Test

Before concluding chapter 3, it is worth a quick analysis of the sensitivity of the model to the market rate of interest $r$ and discounted future value $b$. Because this paper seeks to understand how low interest rate loans and expectations affect the choice of whether to attend college, it is important to have some evaluation as to how sensitive the results are to the assumption used for $b$ and $r$. That is, the
intuition that lowering the interest rate should encourage college attendance ought to hold.

I test the sensitivity by holding the rest of the parameters constant with the same values used in the examples from section 3.2 of \( m_1^w = $30, m_2^w = $32.89, m_1^c = $15.08, m_2^c = $52.92, \) and \( e = $3.4 \). I then test numerous values for variables \( r \) and \( b \), determining if for each set of values the student would choose to attend college borrowing money at the market rate of interest with added cost of tuition or work full-time in period 1. The results of this test are depicted in Table 2.

**Table 2 – Sensitivity Test**

<table>
<thead>
<tr>
<th>Market rate of interest, ( r )</th>
<th>5%</th>
<th>7%</th>
<th>9%</th>
<th>10%</th>
<th>11%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Discounted Future Value, ( b )</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.2</td>
<td>College</td>
<td>College</td>
<td>College</td>
<td>Work</td>
<td>Work</td>
</tr>
<tr>
<td>.5</td>
<td>College</td>
<td>College</td>
<td>College</td>
<td>Work</td>
<td>Work</td>
</tr>
<tr>
<td>.8</td>
<td>College</td>
<td>College</td>
<td>College</td>
<td>Work</td>
<td>Work</td>
</tr>
<tr>
<td><strong>.9333</strong></td>
<td>College</td>
<td>College</td>
<td>College</td>
<td><strong>Work</strong>*</td>
<td>Work</td>
</tr>
<tr>
<td>1</td>
<td>College</td>
<td>College</td>
<td>College</td>
<td>Work</td>
<td>Work</td>
</tr>
</tbody>
</table>

*Values used in sections 3.1 and 3.2; when the interest rate is equal to 10% with a discounted future value of .9333, this student will choose to work full-time instead of attend college when borrowing at the market rate of interest.*

This shows that the model is sensitive to interest rate but not to the discounted future value. This suggests that interest rates will have an effect on the decision to attend college, but the coefficient used for discounted future value will
not. This shows that the model is isolating the interest rate, which is useful in determining the effect of low-interest rate loans on the decision to attend college.

3.4 Summary

In chapter 3, I utilize the model to address two research questions. First, what effect do interest rates and the structure of student loans have on individuals’ incentives to attend college? Second, how do the expectations of graduating high school students about future earnings affect their decision to attend or not attend college?

While not definitive, the treatments show that 1) loan structure, especially low-interest rate loans, can increase the incentive to attend college, and 2) that overly optimistic expectations about future income can mistakenly increase the incentive to attend college. How much either of these does in fact account for cumulative enrollment levels is an open question. What is clear is low-interest student loans and incorrect expectations about future earnings can potentially influence a student to change his or her decision. Further research is clearly needed, but this model is useful in beginning a discussion about the effects of student loan programs and students’ expectations on their decisions on attending college.
Chapter 4: Conclusion

In Chapter 1, I reviewed the present literature on student loan debt. I concluded that while academics and the general public alike proclaim student loan debt to be an important and major issue, very little research exists on the actual affects many loan programs have on the incentives to attend college. Government leaders routinely espouse the virtues of their own individual programs, but rarely are these virtues backed up by hard fact. Further, it is a commonly agreed upon notion that society benefits from a greater share of the population attending college. A better-educated population leads to greater innovation, greater economic growth, and an improved standard of living. However, it is yet unclear that the programs designed to incentivize greater college attendance are actually achieving their stated goal.

In Chapter 2, I presented the basic model by which I sought to examine the two research questions. Ultimately, I believe this model can begin a discussion and further research on the effects of student loan programs.

In Chapter 3, I used the model established in Chapter 2 to test the effects of low interest rate loans and student expectations on the incentives to attend college. I found that both of these loan types do in fact incentivize college attendance and have the potential to change the decision of some students.
In this chapter, I tie together these findings with real world context, before discussing areas for further research.

4.1 Discussion

This paper started with two research questions. First, what effect do interest rates and the structure of student loans have on individuals’ incentives to attend college? Second, how do the expectations of graduating high school students about future earnings affect their decision to attend or not attend college?

In evaluating these research questions, the results of the model were mixed. The model successfully established that low-interest rate loan programs do increase the incentives to attend college. Further, these loans have the potential to change the decision to attend college for some students. This means that for the over 34 million students who took low-interest loans to pay for their undergraduate education, they will gain a greater utility from their college education than without these loans. This is in line with basic economic theory – if these students are rational actors, taking these loans implies a higher utility from the loans than from not taking the loans. Additionally, it is possible that a portion of these students would not have attended college without these loans. However, it is impossible with this simple model to estimate the number of students whose decision was actually affected by the established incentives of a low-interest rate loan.

As for expectations, the model shows how even a small difference between the expected and realized results of attending college for a student can alter their optimal decision. In the example, a $1,080 difference (<2%) between expected and
realized salary was enough to alter the optimal decision. However, while anecdotal evidence and the national conscience suggest that college students have an inflated idea of the value of their college degrees the data simply aren’t there to confirm or deny this trend.

All in all, what the model does is begin an important conversation about how our student loan system actually creates the desired effect of increasing the rate at which students attend college. Politicians want to say their particular programs promote greater college attendance for a portion of the population that would otherwise not attend. What is clear from the model is that this is possible to achieve. However, further research is clearly needed to establish the extent to which certain programs achieve their desired goal. Further, it is unclear which direction students’ misperceptions about future incomes go. What is clear is that in some cases a student may under or overvalue their degree, leading to a suboptimal decision. Again, further research is needed to establish the extent to which this affects our university system and how this potential affect can be combatted.

I conclude with a discussion of some ways the model can be expanded and where further research is necessary in section 4.2. I believe this paper and model to have achieved something and added to the academic literature by paving the way for greater discussion and research into the area of student loan debt.

### 4.2 Areas for Further Research

The most immediately obvious area for further study is into other student loan types. As the cost of education rises, two growing areas of student debt
refinancing are loan forgiveness and income-based repayment. Because both loan forgiveness and income-based repayment require a period of repayment as well as a period of post-repayment to properly address their unique characteristics, anything less than a three period model, such as the two period model I use, is insufficient to analyze these two refinancing methods.

With an expansion of the model to three periods, however, the analysis of all income types could become even clearer. With a first period of work or college attendance, followed by a second period of work (while paying debts for the choice to attend college), with a third period of work for both choices, it is possible to look at how different levels of forgiveness and types of income based repayment plans may incentivize attendance. Such a model is beyond my present capabilities, but this two period model can be used as a starting point in building such a three period model.

Finally, should greater data be made available, perhaps greater examples using real world data could be useful. As it stands, the model builds good theoretical knowledge on the effects of student loan programs, but it is clear in some areas that empirical data don't always match up with theory. To empirically test the predictive power of the model, a substantial amount of data is required. These empirical data extend not only to the costs of college and discount rates of college attendees but also the expectations of potential college students. In my model a single discount rate was used, but it is plausible that someone who does not attend college shows a revealed preference for present consumption greater than those who do attend; this
may have interesting effects on the model. It is also clear expectations can have an important effect, but without adequate data the extent of this effect is unclear.

In conclusion, it is clear that present loan programs do incentivize greater college attendance. What is unclear and requires further research is the extent of this effect and the effect of some of the more unique types of loan options. With more research, it will be possible to create more efficient loan programs that incentivize greater college attendance. As this goal is widely accepted to be a just one and shared by many political leaders, this area of study should be taken seriously and widely studied in the near future.
Appendices
Appendix A – Equations 1, 2, and 8

Equation 1, \( U(C_1, C_2) = C_1^c C_2^b \), is a basic Cobb-Douglas utility function commonly used in microeconomics to show the utility an individual gains from consumption of two goods or baskets of goods, i.e., total consumption in period 1 and total consumption in period 2. Equation 2, \( C_1 + [C_2/(1+r)] = m_1 + [m_2/(1+r)] \), shows total consumption on the left side of the equation equal to total income on the right side of the equation, i.e., an individual consumes in period 1 and period 2 as much but no greater than their income in period 1 and period 2 plus or minus interest from borrowing or lending in period 1. These equations can be found elsewhere, but in my paper come directly from Chapter 4 on utility in Hal Varian’s Intermediate Microeconomics: A modern Approach 8th Edition.

In appendices B-F I show the derivation of equations 3 through 7. In the text, equation 8 is a modification of equation 7 by incorporating a low interest rate for the choice to attend college instead of work full time in period 1, i.e., the student’s period 1 income returns to \( m_1c \) while his period 2 income is now \( m_2 - (1+e)i \).
Appendix B – Equation 3

\[ U(C_1, C_2) = C_1 \ast C_2^b \text{ subject to } C_1 + \left( \frac{C_2}{1 + r} \right) = m_1 + \left( \frac{m_2}{1 + r} \right) \]  \hspace{1cm} (3a)

\[ C_1^* = \left( \frac{1}{1 + b} \right) \left( \frac{m_2}{1 + r} + m_1 \right) \]  \hspace{1cm} (3b)

\[ C_2^* = \left( \frac{b}{1 + b} \right) [m_1(1 + r) + m_2] \]  \hspace{1cm} (3c)

\[ U^* = C_1^* \ast C_2^{*b} \]  \hspace{1cm} (3d)

1. Equation 2: \( C_1 + \frac{C_2}{(1+r)} = m_1 + \frac{m_2}{(1+r)} \)
2. \( C_2(m_1,m_2,C_1,r) = [m_1 + (m_2/(1+r)) - C_1]^*(1+r) \)
3. Equation 1: \( U(C_1, C_2) = C_1 \ast C_2^b \)
4. Substitute \( C_2(m_1,m_2,C_1,r) \) into equation 1: \( U(C_1) = C_1^* \left[ \left( m_1 + (m_2/(1+r)) - C_1 \right)^*(1+r) \right]^b \)
5. Maximize Utility: \( U'(C_1) = \left[ \left( m_1 + (m_2/(1+r)) - C_1 \right)^*(1+r) \right]^b \cdot (1+r) \cdot (b \cdot C_1) \)
   \[ m_1 + (m_2/(1+r)) - C_1 \right)^{b-1} = 0 \]
6. \( C_1^* = \frac{1}{(1+b)} \ast \left( (m_2/1+r) + m_1 \right) \)
7. \( C_2(m_1,m_2,C_1^*,r) = C_2^* = \left( \frac{b}{1+b} \right) \ast \left( (m_1 \ast [1+r] + m_2) \right) \)
8. \( U^* = C_1^* \ast (C_2^*) \)
Appendix C – Equation 4

\[ m_1^c < m_2^c \]  \hspace{1cm} (4a)

\[ m_1^w < m_2^w \]  \hspace{1cm} (4b)

\[ m_1^c < m_1^w \]  \hspace{1cm} (4c)

\[ m_2^w < m_2^c \]  \hspace{1cm} (4d)

\[ m_1^w + m_2^w < m_1^c + m_2^c \]  \hspace{1cm} (4e)

\[ m_1^c < m_1^w < m_2^w < m_2^c \]  \hspace{1cm} (4f)

1. Start with the axioms from equations 4a-4e

2. By the transitive property, if \( m_1^w < m_2^w \) and \( m_2^w < m_2^c \) then \( m_1^w < m_2^c \)

3. By the transitive property, if \( m_1^c < m_1^w \) and \( m_1^w < m_2^w \) then \( m_1^c < m_2^w \)

4. Based on 1-3, it must be that \( m_1^c < m_1^w < m_2^w < m_2^c \)
Appendix D – Equation 5

\[
\text{Max } U = C_1 \ast C_2 \quad \text{subject to } C_1 + \left( \frac{C_2}{1+r} \right) = m_1^w + \left( \frac{m_2^w}{1+r} \right) \quad (5a)
\]

\[
C_1^w = \left( \frac{1}{1+b} \right) \left( \frac{m_2^w}{1+r} + m_1^w \right) \quad (5b)
\]

\[
C_2^w = \left( \frac{b}{1+b} \right) \left[ m_1^w (1+r) + m_2^w \right] \quad (5c)
\]

\[
U^w = C_1^w \ast C_2^w \quad (5d)
\]

1. Equation 2: \( C_1 + \left( \frac{C_2}{1+r} \right) = m_1 + \left[ m_2/(1+r) \right] \)

2. \( C_2(m_1^w, m_2^w, C_1, r) = [m_1^w + (m_2^w/(1+r)) - C_1] \ast (1+r) \)

3. Equation 1: \( U(C_1, C_2) = C_1 \ast C_2^b \)

4. Substitute \( C_2(m_1^w, m_2^w, C_1, r) \) into equation 1: \( U(C_1) = C_1 \ast \left[ m_1^w + (m_2^w/(1+r)) - C_1 \right] \ast (1+r) \)

5. Maximize Utility: \( U'(C_1) = \left[ m_1^w + (m_2^w/(1+r)) - C_1 \right] \ast (1+r) \ast (1+r) \ast (b \ast C_1) \)

\[
\left[ m_1^w + (m_2^w/(1+r)) - C_1 \right] \ast (1+r) \ast (1+r) \ast (b \ast C_1) - 0 = 0
\]

6. \( C_1^w = \frac{1}{1+b} \ast \left[ (m_2^w/1+r) + m_1^w \right] \)

7. \( C_2(m_1, m_2, C_1, r) = \frac{C_2^w}{b/(1+b)} = \left[ (m_1^w \ast [1+r]) + m_2^w \right] \)

8. \( U^w = C_1^w \ast (C_2^w)^b \)
Appendix E – Equation 6

\[ \text{Max } U = C_1 \times C_2^b \text{ subject to } C_1 + \left( \frac{C_2}{1+r} \right) = m_1^w + \left( \frac{m_2^w}{1+r} \right) \]  

(6a)

\[ C_1^c = \left( \frac{1}{1+b} \right) \left( \frac{m_2^c}{1+r} + m_1^c \right) \]  

(6b)

\[ C_2^c = \left( \frac{b}{1+b} \right) \left[ m_1 (1+r) + m_2^c \right] \]  

(6c)

\[ U^c = C_1^c \times C_2^c^b \]  

(6d)

1. Equation 2: \( C_1 + [C_2/(1+r)] = m_1 + [m_2/(1+r)] \)

2. \( C_2(m_1^c,m_2^c,C_1) = [m_1^c + (m_2^c/(1+r))] - C_1 \times (1+r) \)

3. Equation 1: \( U(C_1, C_2) = C_1 \times C_2^b \)

4. Substitute \( C_2(m_1^c,m_2^c,C_1) \) into equation 1: \( U(C_2) = C_1 \times [\{ m_1^c + (m_2^c/(1+r)) - C_1 \} \times (1+r)]^b \)

5. Maximize Utility: \( U'(C_1) = \{ [m_1^c + (m_2^c/(1+r)) - C_1] \times (1+r)]^b \times (1+r) \times (b \times C_1) \}

\[ m_1^c + (m_2^c/(1+r)) - C_1 \} \times (1+r)]^b \times (1+r) \times (b \times C_1) \}

\[ m_1^c + (m_2^c/(1+r)) - C_1 \} \times (1+r)]^b \times (1+r) \times (b \times C_1) \}

6. \( C_1^c = \left( \frac{1}{1+b} \right) \times (m_2^c/1+r) + m_1^c \)

7. \( C_2(m_1,m_2,C_1^c) = C_2^c = \left[ b/(1+b) \right] \times \{ [m_1^c \times (1+r)] + m_2^c \}

8. \( U^c = C_1^c(C_2^c)^b \)
Appendix F – Equation 7

Max $U = C_1 * C_2^b$ subject to $C_1 + \left( \frac{C_2}{1+r} \right) = (m_1^c - e) + \left( \frac{m_2^c}{1+r} \right)$  \hspace{1cm} (7a)

$C_1^e = \left( \frac{1}{1+b} \right) \left[ \frac{m_2^c}{1+r} + (m_1^c - e) \right]$  \hspace{1cm} (7b)

$C_2^e = \left( \frac{b}{1+b} \right) [(m_1^c - e)(1 + r) + m_2^c]$  \hspace{1cm} (7c)

$U^e = C_1^e * C_2^b$  \hspace{1cm} (7d)

1. Equation 2: $C_1 + [C_2/(1+r)] = m_1 + [m_2/(1+r)]$

2. $C_2(m_1^e, m_2^e; C_1, r, e) = [(m_1^e - e) + (m_2^e/(1+r)) - C_1]^* (1+r)$

3. Equation 1: $U(C_1, C_2) = C_1 * C_2^b$

4. Substitute $C_2(m_1^e, m_2^e; C_1, r, e)$ into equation 1: $U(C_1) = C_1 * [(m_1^e - e) + (m_2^e/(1+r)) - C_1]^* (1+r)]^b$

5. Maximize Utility: $U'(C_1) = [(m_1^e - e) + (m_2^e/(1+r)) - C_1]^* (1+r)]^b - (1+r)(b * C_1) [(m_1^e - e) + (m_2^e/(1+r)) - C_1]^* (1+r)]^{b-1} = 0$

6. $C_1^e = [1/(1+b)] * [(m_2^e/1+r) + (m_1^e - e)]$

7. $C_2(m_1, m_2, C_1^e, r, e) = C_2^e = [b/(1+b)] * [(m_1^e - e) * (1+r)] + m_2^e$

8. $U^e = C_1^e (C_2^e)^b$
Citations


